FORMULARIO DE CÁLCULO DIFERENCIAL E INTEGRAL VER.3.3

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VALOR ABSOLUTO

$$|a|=\begin{cases}a & \text{si } a\geq 0\\ -a & \text{si } a<0\end{cases}$$

$|a|=|-a|$

$a\leq|a|$ y $-a\leq|a|$

$|a|\geq 0$ y $|a|=0 \Leftrightarrow a=0$

$|ab|=|a||b|$ ó $\left|\prod_{k=1}^{n}a_k\right|=\prod_{k=1}^{n}|a_k|$

$|a+b|\leq|a|+|b|$ ó $\left|\sum_{k=1}^{n}a_k\right|\leq\sum_{k=1}^{n}|a_k|$

EXPONENTES

$a^p\cdot a^q=a^{p+q}$

$\dfrac{a^p}{a^q}=a^{p-q}$

$(a^p)^q=a^{pq}$

$(a\cdot b)^p=a^p\cdot b^p$

$\left(\dfrac{a}{b}\right)^p=\dfrac{a^p}{b^p}$

$a^{p/q}=\sqrt[q]{a^p}$

LOGARITMOS

$\log_a N=x\Rightarrow a^x=N$

$\log_a MN=\log_a M+\log_a N$

$\log_a\dfrac{M}{N}=\log_a M-\log_a N$

$\log_a M^r=r\log_a M$

$\log_a N=\dfrac{\log_b N}{\log_b a}=\dfrac{\ln N}{\ln a}$

$\log_{10}N=\log N$ y $\log_e N=\ln N$

ALGUNOS PRODUCTOS

$a\cdot(c+d)=ac+ad$

$(a+b)\cdot(a-b)=a^2-b^2$

$(a+b)\cdot(a+b)=(a+b)^2=a^2+2ab+b^2$

$(a-b)\cdot(a-b)=(a-b)^2=a^2-2ab+b^2$

$(x+b)\cdot(x+d)=x^2+(b+d)x+bd$

$(ax+b)\cdot(cx+d)=acx^2+(ad+bc)x+bd$

$(a+b)\cdot(c+d)=ac+ad+bc+bd$

$(a+b)^3=a^3+3a^2b+3ab^2+b^3$

$(a-b)^3=a^3-3a^2b+3ab^2-b^3$

$(a+b+c)^2=a^2+b^2+c^2+2ab+2ac+2bc$

$(a-b)\cdot(a^2+ab+b^2)=a^3-b^3$

$(a-b)\cdot(a^3+a^2b+ab^2+b^3)=a^4-b^4$

$(a-b)\cdot(a^4+a^3b+a^2b^2+ab^3+b^4)=a^5-b^5$

$(a-b)\cdot\left(\sum_{k=1}^{n}a^{n-k}b^{k-1}\right)=a^n-b^n \quad \forall n\in\mathbb{N}$

$(a+b)\cdot(a^2-ab+b^2)=a^3+b^3$

$(a+b)\cdot(a^4-a^3b+a^2b^2-ab^3+b^4)=a^5+b^5$

$(a+b)\cdot\left(\sum_{k=1}^{n}(-1)^{k+1}a^{n-k}b^{k-1}\right)=a^n+b^n \quad \forall 2n-1$

SUMAS Y PRODUCTOS

$a_1+a_2+\cdots+a_n=\sum_{k=1}^{n}a_k$

$\sum_{k=1}^{n}c=nc$

$\sum_{k=1}^{n}ca_k=c\sum_{k=1}^{n}a_k$

$\sum_{k=1}^{n}(a_k+b_k)=\sum_{k=1}^{n}a_k+\sum_{k=1}^{n}b_k$

$\sum_{k=1}^{n}(a_k-a_{k-1})=a_n-a_0$

$\sum_{k=1}^{n}\left[a+(k-1)d\right]=\dfrac{n}{2}\left[2a+(n-1)d\right]$

$=\dfrac{n}{2}(a+l)$

$\sum_{k=1}^{n}ar^{k-1}=a\dfrac{1-r^n}{1-r}=\dfrac{a-rl}{1-r}$

$\sum_{k=1}^{n}k=\dfrac{1}{2}(n^2+n)$

$\sum_{k=1}^{n}k^2=\dfrac{1}{6}(2n^3+3n^2+n)$

$\sum_{k=1}^{n}k^3=\dfrac{1}{4}(n^4+2n^3+n^2)$

$\sum_{k=1}^{n}k^4=\dfrac{1}{30}(6n^5+15n^4+10n^3-n)$

$1+3+5+\cdots+(2n-1)=n^2$

$n!=\prod_{k=1}^{n}k$

$\dbinom{n}{k}=\dfrac{n!}{(n-k)!k!}, \quad k\leq n$

$(x+y)^n=\sum_{k=0}^{n}\dbinom{n}{k}x^{n-k}y^k$

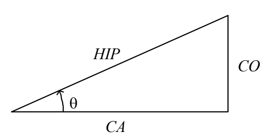
TRIGONOMETRÍA

$\operatorname{sen}\theta=\dfrac{CO}{HIP} \qquad \csc\theta=\dfrac{1}{\operatorname{sen}\theta}$

$\cos\theta=\dfrac{CA}{HIP} \qquad \sec\theta=\dfrac{1}{\cos\theta}$

$\operatorname{tg}\theta=\dfrac{\operatorname{sen}\theta}{\cos\theta}=\dfrac{CO}{CA} \qquad \operatorname{ctg}\theta=\dfrac{1}{\operatorname{tg}\theta}$

π radianes=180°



θ	sen	cos	tg	ctg	sec	csc
0°	0	1	0	∞	1	∞
30°	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$	$\sqrt{3}$	$2/\sqrt{3}$	2
45°	$1/\sqrt{2}$	$1/\sqrt{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$	$1/\sqrt{3}$	2	$2/\sqrt{3}$
90°	1	0	∞	0	∞	1

$y=\angle\operatorname{sen}x \quad y\in\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]$

$y=\angle\cos x \quad y\in[0,\pi]$

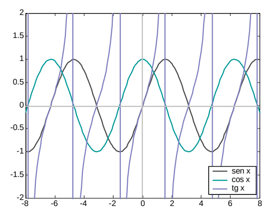
$y=\angle\operatorname{tg}x \quad y\in\left\langle-\dfrac{\pi}{2},\dfrac{\pi}{2}\right\rangle$

$y=\angle\operatorname{ctg}x=\angle\operatorname{tg}\dfrac{1}{x} \quad y\in\langle 0,\pi\rangle$

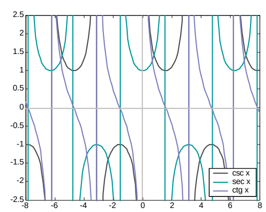
$y=\angle\sec x=\angle\cos\dfrac{1}{x} \quad y\in[0,\pi]$

$y=\angle\csc x=\angle\operatorname{sen}\dfrac{1}{x} \quad y\in\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]$

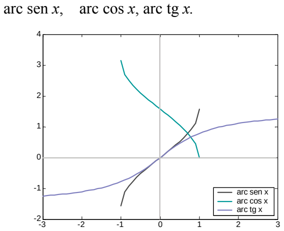
Gráfica 1. Las funciones trigonométricas: sen x, cos x, tg x;.



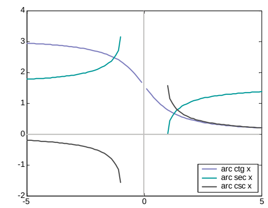
Gráfica 2. Las funciones trigonométricas csc x, sec x, ctg x.



Gráfica 3. Las funciones trigonométricas inversas arc sen x, arc cos x, arc tg x.



Gráfica 4. Las funciones trigonométricas inversas arc ctg x, arc sec x, arc csc x.



IDENTIDADES TRIGONOMÉTRICAS

$\operatorname{sen}^2\theta+\cos^2\theta=1$

$1+\operatorname{ctg}^2\theta=\csc^2\theta$

$\operatorname{tg}^2\theta+1=\sec^2\theta$

$\operatorname{sen}(-\theta)=-\operatorname{sen}\theta$

$\cos(-\theta)=\cos\theta$

$\operatorname{tg}(-\theta)=-\operatorname{tg}\theta$

$\operatorname{sen}(\theta+2\pi)=\operatorname{sen}\theta$

$\cos(\theta+2\pi)=\cos\theta$

$\operatorname{tg}(\theta+2\pi)=\operatorname{tg}\theta$

$\operatorname{sen}(\theta+\pi)=-\operatorname{sen}\theta$

$\cos(\theta+\pi)=-\cos\theta$

$\operatorname{tg}(\theta+\pi)=\operatorname{tg}\theta$

$\operatorname{sen}(\theta+n\pi)=(-1)^n\operatorname{sen}\theta$

$\cos(\theta+n\pi)=(-1)^n\cos\theta$

$\operatorname{tg}(\theta+n\pi)=\operatorname{tg}\theta$

$\operatorname{sen}(n\pi)=0$

$\cos(n\pi)=(-1)^n$

$\operatorname{tg}(n\pi)=0$

$\operatorname{sen}\left(\dfrac{2n+1}{2}\pi\right)=(-1)^n$

$\cos\left(\dfrac{2n+1}{2}\pi\right)=0$

$\operatorname{tg}\left(\dfrac{2n+1}{2}\pi\right)=\infty$

$\operatorname{sen}\theta=\cos\left(\theta-\dfrac{\pi}{2}\right)$

$\cos\theta=\operatorname{sen}\left(\theta+\dfrac{\pi}{2}\right)$

$\operatorname{sen}(\alpha\pm\beta)=\operatorname{sen}\alpha\cos\beta\pm\cos\alpha\operatorname{sen}\beta$

$\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\operatorname{sen}\alpha\operatorname{sen}\beta$

$\operatorname{tg}(\alpha\pm\beta)=\dfrac{\operatorname{tg}\alpha\pm\operatorname{tg}\beta}{1\mp\operatorname{tg}\alpha\operatorname{tg}\beta}$

$\operatorname{sen}2\theta=2\operatorname{sen}\theta\cos\theta$

$\cos 2\theta=\cos^2\theta-\operatorname{sen}^2\theta$

$\operatorname{tg}2\theta=\dfrac{2\operatorname{tg}\theta}{1-\operatorname{tg}^2\theta}$

$\operatorname{sen}^2\theta=\dfrac{1}{2}(1-\cos 2\theta)$

$\cos^2\theta=\dfrac{1}{2}(1+\cos 2\theta)$

$\operatorname{tg}^2\theta=\dfrac{1-\cos 2\theta}{1+\cos 2\theta}$

$\operatorname{sen}\alpha+\operatorname{sen}\beta=2\operatorname{sen}\dfrac{1}{2}(\alpha+\beta)\cdot\cos\dfrac{1}{2}(\alpha-\beta)$

$\operatorname{sen}\alpha-\operatorname{sen}\beta=2\operatorname{sen}\dfrac{1}{2}(\alpha-\beta)\cdot\cos\dfrac{1}{2}(\alpha+\beta)$

$\cos\alpha+\cos\beta=2\cos\dfrac{1}{2}(\alpha+\beta)\cdot\cos\dfrac{1}{2}(\alpha-\beta)$

$\cos\alpha-\cos\beta=-2\operatorname{sen}\dfrac{1}{2}(\alpha+\beta)\cdot\operatorname{sen}\dfrac{1}{2}(\alpha-\beta)$

$\operatorname{tg}\alpha\pm\operatorname{tg}\beta=\dfrac{\operatorname{sen}(\alpha\pm\beta)}{\cos\alpha\cdot\cos\beta}$

$\operatorname{sen}\alpha\cdot\cos\beta=\dfrac{1}{2}\left[\operatorname{sen}(\alpha-\beta)+\operatorname{sen}(\alpha+\beta)\right]$

$\operatorname{sen}\alpha\cdot\operatorname{sen}\beta=\dfrac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$

$\cos\alpha\cdot\cos\beta=\dfrac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$

$\operatorname{tg}\alpha\cdot\operatorname{tg}\beta=\dfrac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{\operatorname{ctg}\alpha+\operatorname{ctg}\beta}$

FUNCIONES HIPERBÓLICAS

$\operatorname{senh}x=\dfrac{e^x-e^{-x}}{2}$

$\cosh x=\dfrac{e^x+e^{-x}}{2}$

$\operatorname{tgh}x=\dfrac{\operatorname{senh}x}{\cosh x}=\dfrac{e^x-e^{-x}}{e^x+e^{-x}}$

$\operatorname{ctgh}x=\dfrac{1}{\operatorname{tgh}x}=\dfrac{e^x+e^{-x}}{e^x-e^{-x}}$

$\operatorname{sech}x=\dfrac{1}{\cosh x}=\dfrac{2}{e^x+e^{-x}}$

$\operatorname{csch}x=\dfrac{1}{\operatorname{senh}x}=\dfrac{2}{e^x-e^{-x}}$

$\operatorname{senh}:\mathbb{R}\to\mathbb{R}$

$\cosh:\mathbb{R}\to[1,\infty\rangle$

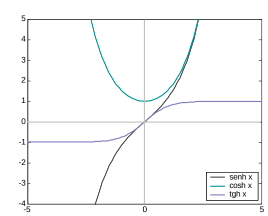
$\operatorname{tgh}:\mathbb{R}\to\langle-1,1\rangle$

$\operatorname{ctgh}:\mathbb{R}-\{0\}\to\langle-\infty,-1\rangle\cup\langle 1,\infty\rangle$

$\operatorname{sech}:\mathbb{R}\to\langle 0,1]$

$\operatorname{csch}:\mathbb{R}-\{0\}\to\mathbb{R}-\{0\}$

Gráfica 5. Las funciones hiperbólicas senh x, cosh x, tgh x.



FUNCS HIPERBÓLICAS INVERSAS

$\operatorname{senh}^{-1}x=\ln\left(x+\sqrt{x^2+1}\right), \quad \forall x\in\mathbb{R}$

$\cosh^{-1}x=\ln\left(x\pm\sqrt{x^2-1}\right), \quad x\geq 1$

$\operatorname{tgh}^{-1}x=\dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right), \quad |x|<1$

$\operatorname{ctgh}^{-1}x=\dfrac{1}{2}\ln\left(\dfrac{x+1}{x-1}\right), \quad |x|>1$

$\operatorname{sech}^{-1}x=\ln\left(\dfrac{1\pm\sqrt{1-x^2}}{x}\right), \quad 0<x\leq 1$

$\operatorname{csch}^{-1}x=\ln\left(\dfrac{1}{x}+\dfrac{\sqrt{x^2+1}}{|x|}\right), \quad x\neq 0$

IDENTIDADES DE FUNCS HIP

$$\cosh^2 x - \operatorname{senh}^2 x = 1$$
$$1 - \operatorname{tgh}^2 x = \operatorname{sech}^2 x$$
$$\operatorname{ctgh}^2 x - 1 = \operatorname{csch} x$$
$$\operatorname{senh}(-x) = -\operatorname{senh} x$$
$$\cosh(-x) = \cosh x$$
$$\operatorname{tgh}(-x) = -\operatorname{tgh} x$$
$$\operatorname{senh}(x \pm y) = \operatorname{senh} x \cosh y \pm \cosh x \operatorname{senh} y$$
$$\cosh(x \pm y) = \cosh x \cosh y \pm \operatorname{senh} x \operatorname{senh} y$$
$$\operatorname{tgh}(x \pm y) = \frac{\operatorname{tgh} x \pm \operatorname{tgh} y}{1 \pm \operatorname{tgh} x \operatorname{tgh} y}$$
$$\operatorname{senh} 2x = 2 \operatorname{senh} x \cosh x$$
$$\cosh 2x = \cosh^2 x + \operatorname{senh}^2 x$$
$$\operatorname{tgh} 2x = \frac{2 \operatorname{tgh} x}{1 + \operatorname{tgh}^2 x}$$
$$\operatorname{senh}^2 x = \frac{1}{2}(\cosh 2x - 1)$$
$$\cosh^2 x = \frac{1}{2}(\cosh 2x + 1)$$
$$\operatorname{tgh}^2 x = \frac{\cosh 2x - 1}{\cosh 2x + 1}$$
$$\operatorname{tgh} x = \frac{\operatorname{senh} 2x}{\cosh 2x + 1}$$

OTRAS

$$Ax^2 + Bx + C = 0$$
$$\Rightarrow x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$
$$B^2 - 4AC = \text{discriminante}$$

LÍMITES

$$\lim_{x \to 0} (1+x)^{\frac{1}{x}} = e = 2.71828...$$
$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$$
$$\lim_{x \to 0} \frac{\operatorname{sen} x}{x} = 1$$
$$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$
$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$
$$\lim_{x \to 1} \frac{x - 1}{\ln x} = 1$$

DERIVADAS

$$D_x f(x) = \frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$
$$\frac{d}{dx}(c) = 0$$
$$\frac{d}{dx}(cx) = c$$
$$\frac{d}{dx}(cx^n) = ncx^{n-1}$$
$$\frac{d}{dx}(u \pm v \pm w \pm \cdots) = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx} \pm \cdots$$
$$\frac{d}{dx}(cu) = c\frac{du}{dx}$$
$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$
$$\frac{d}{dx}(uvw) = uv\frac{dw}{dx} + uw\frac{dv}{dx} + vw\frac{du}{dx}$$
$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v(du/dx) - u(dv/dx)}{v^2}$$
$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$$

$$\frac{dF}{dx} = \frac{dF}{du} \cdot \frac{du}{dx} \quad \text{(Regla de la Cadena)}$$
$$\frac{du}{dx} = \frac{1}{dx/du}$$
$$\frac{dF}{dx} = \frac{dF/du}{dx/du}$$
$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{f_2'(t)}{f_1'(t)} \quad \text{donde} \begin{cases} x = f_1(t) \\ y = f_2(t) \end{cases}$$

DERIVADA DE FUNCS LOG & EXP

$$\frac{d}{dx}(\ln u) = \frac{du/dx}{u} = \frac{1}{u} \cdot \frac{du}{dx}$$
$$\frac{d}{dx}(\log u) = \frac{\log e}{u} \cdot \frac{du}{dx}$$
$$\frac{d}{dx}(\log_a u) = \frac{\log_a e}{u} \cdot \frac{du}{dx} \quad a > 0,\ a \neq 1$$
$$\frac{d}{dx}(e^u) = e^u \cdot \frac{du}{dx}$$
$$\frac{d}{dx}(a^u) = a^u \ln a \cdot \frac{du}{dx}$$
$$\frac{d}{dx}(u^v) = vu^{v-1}\frac{du}{dx} + \ln u \cdot u^v \cdot \frac{dv}{dx}$$

DERIVADA DE FUNCIONES TRIGO

$$\frac{d}{dx}(\operatorname{sen} u) = \cos u \frac{du}{dx}$$
$$\frac{d}{dx}(\cos u) = -\operatorname{sen} u \frac{du}{dx}$$
$$\frac{d}{dx}(\operatorname{tg} u) = \sec^2 u \frac{du}{dx}$$
$$\frac{d}{dx}(\operatorname{ctg} u) = -\csc^2 u \frac{du}{dx}$$
$$\frac{d}{dx}(\sec u) = \sec u \operatorname{tg} u \frac{du}{dx}$$
$$\frac{d}{dx}(\csc u) = -\csc u \operatorname{ctg} u \frac{du}{dx}$$
$$\frac{d}{dx}(\operatorname{vers} u) = \operatorname{sen} u \frac{du}{dx}$$

DERIV DE FUNCS TRIGO INVER

$$\frac{d}{dx}(\angle \operatorname{sen} u) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$
$$\frac{d}{dx}(\angle \cos u) = -\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$
$$\frac{d}{dx}(\angle \operatorname{tg} u) = \frac{1}{1+u^2} \cdot \frac{du}{dx}$$
$$\frac{d}{dx}(\angle \operatorname{ctg} u) = -\frac{1}{1+u^2} \cdot \frac{du}{dx}$$
$$\frac{d}{dx}(\angle \sec u) = \pm \frac{1}{u\sqrt{u^2-1}} \cdot \frac{du}{dx} \begin{cases} + \text{ si } u > 1 \\ - \text{ si } u < -1 \end{cases}$$
$$\frac{d}{dx}(\angle \csc u) = \mp \frac{1}{u\sqrt{u^2-1}} \cdot \frac{du}{dx} \begin{cases} - \text{ si } u > 1 \\ + \text{ si } u < -1 \end{cases}$$
$$\frac{d}{dx}(\angle \operatorname{vers} u) = \frac{1}{\sqrt{2u-u^2}} \cdot \frac{du}{dx}$$

DERIVADA DE FUNCS HIPERBÓLICAS

$$\frac{d}{dx}\operatorname{senh} u = \cosh u \frac{du}{dx}$$
$$\frac{d}{dx}\cosh u = \operatorname{senh} u \frac{du}{dx}$$
$$\frac{d}{dx}\operatorname{tgh} u = \operatorname{sech}^2 u \frac{du}{dx}$$
$$\frac{d}{dx}\operatorname{ctgh} u = -\operatorname{csch}^2 u \frac{du}{dx}$$
$$\frac{d}{dx}\operatorname{sech} u = -\operatorname{sech} u \operatorname{tgh} u \frac{du}{dx}$$
$$\frac{d}{dx}\operatorname{csch} u = -\operatorname{csch} u \operatorname{ctgh} u \frac{du}{dx}$$

DERIVADA DE FUNCS HIP INV

$$\frac{d}{dx}\operatorname{senh}^{-1} u = \frac{1}{\sqrt{1+u^2}} \cdot \frac{du}{dx}$$
$$\frac{d}{dx}\cosh^{-1} u = \frac{\pm 1}{\sqrt{u^2-1}} \cdot \frac{du}{dx},\ u > 1 \begin{cases} + \text{ si } \cosh^{-1} u > 0 \\ - \text{ si } \cosh^{-1} u < 0 \end{cases}$$
$$\frac{d}{dx}\operatorname{tgh}^{-1} u = \frac{1}{1-u^2} \cdot \frac{du}{dx},\ |u| < 1$$
$$\frac{d}{dx}\operatorname{ctgh}^{-1} u = \frac{1}{1-u^2} \cdot \frac{du}{dx},\ |u| > 1$$
$$\frac{d}{dx}\operatorname{sech}^{-1} u = \frac{\mp 1}{u\sqrt{1-u^2}} \cdot \frac{du}{dx} \begin{cases} - \text{ si } \operatorname{sech}^{-1} u > 0, u \in \langle 0,1 \rangle \\ + \text{ si } \operatorname{sech}^{-1} u < 0, u \in \langle 0,1 \rangle \end{cases}$$
$$\frac{d}{dx}\operatorname{csch}^{-1} u = -\frac{1}{|u|\sqrt{1+u^2}} \cdot \frac{du}{dx},\ u \neq 0$$

INTEGRALES DEFINIDAS, PROPIEDADES

$$\int_a^b \{f(x) \pm g(x)\} dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$
$$\int_a^b cf(x)dx = c \cdot \int_a^b f(x)dx \quad c \in \mathbb{R}$$
$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$
$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$
$$\int_a^a f(x)dx = 0$$
$$m \cdot (b-a) \leq \int_a^b f(x)dx \leq M \cdot (b-a)$$
$$\Leftrightarrow m \leq f(x) \leq M \ \forall x \in [a,b],\ m, M \in \mathbb{R}$$
$$\int_a^b f(x)dx \leq \int_a^b g(x)dx$$
$$\Leftrightarrow f(x) \leq g(x) \ \forall x \in [a,b]$$
$$\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx \text{ si } a < b$$

INTEGRALES

$$\int a\,dx = ax$$
$$\int af(x)dx = a\int f(x)dx$$
$$\int (u \pm v \pm w \pm \cdots)dx = \int udx \pm \int vdx \pm \int wdx \pm \cdots$$
$$\int udv = uv - \int vdu \quad \text{(Integración por partes)}$$
$$\int u^n du = \frac{u^{n+1}}{n+1} \quad n \neq -1$$
$$\int \frac{du}{u} = \ln|u|$$

INTEGRALES DE FUNCS LOG & EXP

$$\int e^u du = e^u$$
$$\int a^u du = \frac{a^u}{\ln a} \begin{cases} a > 0 \\ a \neq 1 \end{cases}$$
$$\int ua^u du = \frac{a^u}{\ln a} \cdot \left(u - \frac{1}{\ln a}\right)$$
$$\int ue^u du = e^u(u-1)$$
$$\int \ln u\,du = u \ln u - u = u(\ln u - 1)$$
$$\int \log_a u\,du = \frac{1}{\ln a}(u \ln u - u) = \frac{u}{\ln a}(\ln u - 1)$$
$$\int u \log_a u\,du = \frac{u^2}{4} \cdot (2\log_a u - 1)$$
$$\int u \ln u\,du = \frac{u^2}{4}(2\ln u - 1)$$

INTEGRALES DE FUNCS TRIGO

$$\int \operatorname{sen} u\,du = -\cos u$$
$$\int \cos u\,du = \operatorname{sen} u$$
$$\int \sec^2 u\,du = \operatorname{tg} u$$
$$\int \csc^2 u\,du = -\operatorname{ctg} u$$
$$\int \sec u \operatorname{tg} u\,du = \sec u$$
$$\int \csc u \operatorname{ctg} u\,du = -\csc u$$
$$\int \operatorname{tg} u\,du = -\ln \cos u = \ln \sec u$$
$$\int \operatorname{ctg} u\,du = \ln \operatorname{sen} u$$
$$\int \sec u\,du = \ln(\sec u + \operatorname{tg} u)$$
$$\int \csc u\,du = \ln(\csc u - \operatorname{ctg} u)$$
$$\int \operatorname{sen}^2 u\,du = \frac{u}{2} - \frac{1}{4}\operatorname{sen} 2u$$
$$\int \cos^2 u\,du = \frac{u}{2} + \frac{1}{4}\operatorname{sen} 2u$$
$$\int \operatorname{tg}^2 u\,du = \operatorname{tg} u - u$$
$$\int \operatorname{ctg}^2 u\,du = -(\operatorname{ctg} u + u)$$
$$\int u \operatorname{sen} u\,du = \operatorname{sen} u - u \cos u$$
$$\int u \cos u\,du = \cos u + u \operatorname{sen} u$$

INTEGRALES DE FUNCS TRIGO INV

$$\int \angle \operatorname{sen} u\,du = u\angle \operatorname{sen} u + \sqrt{1-u^2}$$
$$\int \angle \cos u\,du = u\angle \cos u - \sqrt{1-u^2}$$
$$\int \angle \operatorname{tg} u\,du = u\angle \operatorname{tg} u - \ln\sqrt{1+u^2}$$
$$\int \angle \operatorname{ctg} u\,du = u\angle \operatorname{ctg} u + \ln\sqrt{1+u^2}$$
$$\int \angle \sec u\,du = u\angle \sec u - \ln\left(u + \sqrt{u^2-1}\right)$$
$$= u\angle \sec u - \angle \cosh u$$
$$\int \angle \csc u\,du = u\angle \csc u + \ln\left(u + \sqrt{u^2-1}\right)$$
$$= u\angle \csc u + \angle \cosh u$$

INTEGRALES DE FUNCS HIP

$$\int \operatorname{senh} u\,du = \cosh u$$
$$\int \cosh u\,du = \operatorname{senh} u$$
$$\int \operatorname{sech}^2 u\,du = \operatorname{tgh} u$$
$$\int \operatorname{csch}^2 u\,du = -\operatorname{ctgh} u$$
$$\int \operatorname{sech} u \operatorname{tgh} u\,du = -\operatorname{sech} u$$
$$\int \operatorname{csch} u \operatorname{ctgh} u\,du = -\operatorname{csch} u$$
$$\int \operatorname{tgh} u\,du = \ln \cosh u$$
$$\int \operatorname{ctgh} u\,du = \ln|\operatorname{senh} u|$$
$$\int \operatorname{sech} u\,du = \angle \operatorname{tg}(\operatorname{senh} u)$$
$$\int \operatorname{csch} u\,du = -\operatorname{ctgh}^{-1}(\cosh u)$$
$$= \ln \operatorname{tgh}\frac{1}{2}u$$

INTREGRALES DE FRAC

$$\int \frac{du}{u^2+a^2} = \frac{1}{a}\angle \operatorname{tg}\frac{u}{a}$$
$$= -\frac{1}{a}\angle \operatorname{ctg}\frac{u}{a}$$
$$\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\frac{u-a}{u+a} \quad (u^2 > a^2)$$
$$\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\frac{a+u}{a-u} \quad (u^2 < a^2)$$

INTEGRALES CON $\sqrt{\ }$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \angle \operatorname{sen}\frac{u}{a}$$
$$= -\angle \cos\frac{u}{a}$$
$$\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln\left(u + \sqrt{u^2 \pm a^2}\right)$$
$$\int \frac{du}{u\sqrt{a^2 \pm u^2}} = \frac{1}{a}\ln\left|\frac{u}{a + \sqrt{a^2 \pm u^2}}\right|$$
$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\angle \cos\frac{a}{u}$$
$$= \frac{1}{a}\angle \sec\frac{u}{a}$$
$$\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\angle \operatorname{sen}\frac{u}{a}$$
$$\int \sqrt{u^2 \pm a^2}\,du = \frac{u}{2}\sqrt{u^2 \pm a^2} \pm \frac{a^2}{2}\ln\left(u + \sqrt{u^2 \pm a^2}\right)$$

MAS INTEGRALES

$$\int e^{au} \operatorname{sen} bu\,du = \frac{e^{au}(a \operatorname{sen} bu - b \cos bu)}{a^2+b^2}$$
$$\int e^{au} \cos bu\,du = \frac{e^{au}(a \cos bu + b \operatorname{sen} bu)}{a^2+b^2}$$

ALGUNAS SERIES

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)(x-x_0)^2}{2!} + \cdots + \frac{f^{(n)}(x_0)(x-x_0)^n}{n!} \quad \text{: Taylor}$$
$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \cdots + \frac{f^{(n)}(0)x^n}{n!} \quad \text{: Maclaurin}$$
$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$
$$\operatorname{sen} x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + (-1)^{n-1}\frac{x^{2n-1}}{(2n-1)!}$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + (-1)^{n-1}\frac{x^{2n-2}}{(2n-2)!}$$
$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n-1}\frac{x^n}{n}$$
$$\angle \operatorname{tg} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots + (-1)^{n-1}\frac{x^{2n-1}}{2n-1}$$